

# Comment on the recent COMPASS data on the spin structure function $g_1$

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We examine the recent COMPASS data on the spin structure function  $g_1$  singlet. We show that it is rather difficult to use the data in the present form in order to draw conclusions on the initial parton densities. However, our tentative estimate is that the data better agree with positive rather than negative initial gluon densities.

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## I. INTRODUCTION

The COMPASS collaboration has recently presented new data, Ref. [1], on the singlet component of the spin structure function  $g_1$ . These data were obtained from the measurements of the longitudinal spin asymmetries in the scattering of muons off the *LiD* target. They found that approximately

$$g_1(x, Q^2) = 0, \quad (1)$$

with small errors, in a wide region of  $x$  and at small  $Q^2$ . More precisely, the kinematic region covered in Ref. [1] is

$$G_{COMPASS} : \quad 10^{-4} \lesssim x \lesssim 10^{-1}; \quad 10^{-1} \text{ GeV}^2 \lesssim Q^2 \lesssim 1 \text{ GeV}^2. \quad (2)$$

The fact that  $g_1$  is zero in the wide region of  $x$  at the first sight looks quite unexpected and even intriguing, and clearly requires a theoretical explanation. The most-known theoretical tool for describing  $g_1$  is the Standard Approach (SA), based on the DGLAP evolution equations, Ref. [2], combined with the standard fits, Refs. [3] -[5], for the initial parton densities. However, the values of  $Q^2$  in the region  $G_{COMPASS}$  are quite small and therefore this region is beyond the reach of DGLAP. In Refs. [6, 7] we have suggested an alternative approach for describing  $g_1$  at small  $x$  and arbitrary  $Q^2$ . Briefly, it combines the total resummation of the leading logarithms of  $x$  suggested in Refs. [8, 9] with the shift

$$Q^2 \rightarrow \bar{Q}^2 = Q^2 + \mu^2, \quad (3)$$

with  $\mu \approx 5.5 \text{ GeV}$  for the singlet  $g_1$ . This shift automatically leads to an effective change of  $x$ :

$$x \rightarrow \bar{x} = x + z \quad (4)$$

where  $z = \mu^2/w$ ,  $w \equiv 2pq$ , and  $p, q$  are the proton and virtual photon momenta respectively. The shifts of  $Q^2$  and  $x$  in Eqs. (3,4) allowed us to express  $g_1$  at small  $x$  and arbitrary  $Q^2$  in terms of  $g_1^{LL}(x, Q^2)$  obtained in Refs. [9] by the total resummation of the leading logarithmic contributions in the region of small  $x$  and large  $Q^2$ :  $g_1$  at small  $x$  and arbitrary  $Q^2$  can be written as  $g_1^{LL}(\bar{x}, \bar{Q}^2)$ . Let us notice that introducing a shift, similarly to Eq. (3), has been a common tool for describing the small- $Q^2$  kinematic region, see e.g. Refs. [10] and refs. therein. However, contrary to all other approaches, we have introduced the shift Eqs. (3,4) from the analysis of the Feynman graphs involved, and the value of  $\mu$  is fixed from theoretical considerations (see Refs. [6, 7] for detail). In Ref. [6] we have predicted that  $g_1$  should not depend on  $x$  in the COMPASS kinematic region: indeed Eq. (4) shows that  $g_1$  depends on  $z$  rather than  $x$  in the region  $G_{COMPASS}$ . The exact value of  $g_1$  in this region cannot be predicted because it strongly depends on the interplay between the quark and gluon contributions. Those contributions involve the coefficient functions and the initial quark and gluon densities  $\delta q$  and  $\delta g$  which are unknown. On the other hand, the very fact that  $g_1 = 0$  approximately, can be used to estimate  $\delta q$  and  $\delta g$ . A straightforward application of our results in Refs. [6, 7] to the region  $G_{COMPASS}$  might be misleading. The point is that the approach of Refs. [6, 7] is valid in the kinematic region of small  $\bar{x}$ , i.e. for  $z \ll 1$ , whereas in the COMPASS experiments  $30 \text{ GeV}^2 \lesssim w \lesssim 270 \text{ GeV}^2$  and therefore

$$1 \lesssim z \lesssim 0.1, \quad (5)$$

In order to extend the approach of Refs. [6, 7] to the region of Eq. (5) we suggest in the present paper a simple interpolation expression for  $g_1$  which combines the approach of Refs. [6, 7] with accounting for the non-logarithmic contributions to the coefficient functions in the fixed orders in  $\alpha_s$ .

The present paper is organized as follows: in Sect. II we remind the expressions for  $g_1$  obtained in Ref. [6]; in Sect. III we generalize them to the region of Eq. (5), adding non-logarithmic contributions to the coefficient functions and anomalous dimensions. Then, in Sect. IV we apply this technique to the COMPASS data. Sect. V is for our concluding remarks.

## II. EXPRESSION FOR THE SINGLET $g_1$ AT SMALL $\bar{x}$ AND ARBITRARY $Q^2$ IN THE LEADING LOGARITHMIC APPROXIMATION

Explicit expressions for the singlet  $g_1$  at small  $\bar{x}$  and arbitrary  $Q^2$  in the Leading Logarithmic Approximation (LLA) were obtained in Refs. [6, 7]. They account for the total resummation of DL contributions and for the running  $\alpha_s$  effects. According to Ref. [6], the LLA expression ( $\equiv g_1^{LL}$ ) for the singlet  $g_1$  at small  $x$  and arbitrary  $Q^2$  is:

$$g_1^{LL}(\bar{x}, \bar{Q}^2) = \frac{\langle e_q^2 \rangle}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^\omega \times \\ \times \left[ \left( C_q^{(+)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega_{(+)}(\omega)} + C_q^{(-)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega_{(-)}(\omega)} \right) \delta q(\omega) + \right. \\ \left. + \left( C_g^{(+)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega_{(+)}(\omega)} + C_g^{(-)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega_{(-)}(\omega)} \right) \delta g(\omega) \right]. \quad (6)$$

The coefficient functions  $C_q^{(\pm)}(\omega)$  and  $C_g^{(\pm)}(\omega)$  as well as the exponents  $\Omega_{(\pm)}(\omega)$  are expressed through the anomalous dimensions  $H_{ik}$  which account for the total resummation of DL contributions and the running  $\alpha_s$  effects, as follows. We remind here that in our approach we do not use the DGLAP parametrization  $\alpha_s = \alpha_s(Q^2)$ . Instead, we use the alternative we suggested in Ref. [13]. It allows us to consider the region of really small  $Q^2$  and at the same time to be within the framework of the Perturbative QCD.

### A. Expressions for the exponents $\Omega_{(\pm)}$

The explicit expressions for  $\Omega_{(\pm)}$  are:

$$\Omega_{(\pm)} = \frac{1}{2} [H_{qq} + H_{gg} \pm \sqrt{R}], \quad (7)$$

where

$$R = (H_{qq} - H_{gg})^2 + 4H_{qg}H_{gq} \quad (8)$$

and  $H_{qq}, H_{gq}, H_{qg}, H_{gg}$  are the anomalous dimensions calculated in LLA.

### B. Coefficient functions

The expressions for the coefficient functions are also written in terms of  $H_{ik}$ :

$$C_q^{(+)} = \frac{\omega(-X + \sqrt{R})}{2T\sqrt{R}}, \quad C_g^{(+)} = \frac{\omega H_{qg}}{T\sqrt{R}}, \\ C_q^{(-)} = \frac{\omega(X + \sqrt{R})}{2T\sqrt{R}}, \quad C_g^{(-)} = -\frac{\omega H_{qg}}{T\sqrt{R}}. \quad (9)$$

Here

$$X = H_{gg} - H_{qq}, \quad T = \omega^2 - \omega(H_{gg} + H_{qq}) + (H_{gg}H_{qq} - H_{gq}H_{qg}) \quad (10)$$

### C. Anomalous dimensions

Here we have:

$$\begin{aligned} H_{qq} &= \frac{1}{2} \left[ \omega - Z + \frac{b_{qq} - b_{gg}}{Z} \right], & H_{qg} &= \frac{b_{qg}}{Z}, \\ H_{gg} &= \frac{1}{2} \left[ \omega - Z - \frac{b_{qq} - b_{gg}}{Z} \right], & H_{gq} &= \frac{b_{gq}}{Z} \end{aligned} \quad (11)$$

where

$$Z = \frac{1}{\sqrt{2}} \sqrt{(\omega^2 - 2(b_{qq} + b_{gg})) + \sqrt{(\omega^2 - 2(b_{qq} + b_{gg}))^2 - 4(b_{qq} - b_{gg})^2 - 16b_{gq}b_{qg}}}, \quad (12)$$

$$b_{ik} = a_{ik} + V_{ik}, \quad (13)$$

with the Born contributions  $a_{ik}$  defined as follows:

$$a_{qq} = \frac{A(\omega)C_F}{2\pi}, \quad a_{qg} = \frac{A'(\omega)C_F}{\pi}, \quad a_{gq} = -\frac{n_f A'(\omega)}{2\pi}, \quad a_{gg} = \frac{4NA(\omega)}{2\pi}, \quad (14)$$

$$V_{ik} = \frac{m_{ik}}{\pi^2} D(\omega), \quad (15)$$

$$m_{qq} = \frac{C_F}{2N}, \quad m_{gg} = -2N^2, \quad m_{gq} = n_f \frac{N}{2}, \quad m_{qg} = -NC_F, \quad (16)$$

and

$$A(\omega) = \frac{1}{b} \left[ \frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho + \eta)^2 + \pi^2} \right], \quad (17)$$

$$A'(\omega) = \frac{1}{b} \left[ \frac{1}{\eta} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho + \eta)^2} \right], \quad (18)$$

$$D(\omega) = \frac{1}{2b^2} \int_0^\infty d\rho e^{-\omega\rho} \ln((\rho + \eta)/\eta) \left[ \frac{\rho + \eta}{(\rho + \eta)^2 + \pi^2} + \frac{1}{\rho + \eta} \right], \quad (19)$$

with  $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$  and  $b = (33 - 2n_f)/(12\pi)$ .

### III. EXPRESSION FOR THE SINGLET $g_1$ AT ARBITRARY $\bar{x}$ AND $Q^2$

Our goal now is to obtain explicit expressions for the singlet  $g_1$  which could be valid at arbitrary  $\bar{x}$  and  $Q^2$ . The point is that the Eqs. (9) and (11) for the coefficient functions and anomalous dimensions present the total resummation of the leading logarithms of  $\bar{x}$  but those contributions are large when  $\bar{x} \ll 1$  only. Alternatively, non-logarithmic contributions can be large at large  $\bar{x} \lesssim 1$  and should be taken into account at large  $\bar{x}$ . Such terms are beyond the rich of our approach, so we cannot do the total resummation of them. Instead, we can obtain them in the orders  $\sim \alpha_s$  and  $\sim \alpha_s^2$ . Adding these contributions to the expressions in Eqs. (9) and (11), we arrive at new formulae for the coefficient functions and anomalous dimensions, which are valid at arbitrary  $\bar{x}$ . In doing so, we can use the DGLAP results for the anomalous dimensions and coefficient functions. Let us demonstrate it in detail, using an example of the singlet coefficient function  $C_q$ . The dealing with the other coefficient function and anomalous dimensions is quite similar. The NLO DGLAP singlet coefficient function  $C_q^{DGLAP}$  in the  $\omega$ -space and at integer  $\omega = n$  is (see e.g. [11])

$$C_q^{DGLAP} = 1 + \frac{\alpha_s(Q^2)C_F}{2\pi} \left[ -S_2(n) + (S_1)^2(n) + \left( \frac{3}{2} - \frac{1}{n(n+1)} \right) S_1(n) + \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{n+1} - \frac{9}{2} \right] \quad (20)$$

where we use the standard notations

$$S_1(n) = \sum_{k=1}^n 1/k, \quad S_2(n) = \sum_{k=1}^n 1/k^2. \quad (21)$$

The expression (20) is obtained by direct calculation of the Feynman graphs and is insensitive to the value of  $Q^2$ , save the parametrization of  $\alpha_s$ . So, we can borrow it, though after some appropriate changes: In the first place it should be valid at arbitrary  $\omega$ ; second, according to the results of Ref. [13], the coupling  $\alpha_s(Q^2)$  should be changed to  $A(\omega)$  defined in Eq. (17). The analytic continuation of Eq. (20) to arbitrary  $\omega$  is obtained through expressing the sums in Eq. (21) in terms of the polygamma  $\psi$ -function and the Euler constant  $\mathbf{C}$ :

$$S_1(n) = \mathbf{C} + n\psi(n-1), \quad S_2(n-1) = \frac{\pi^2}{6} + \psi'(n). \quad (22)$$

After that we obtain an expression which we address as  $C_q^{(1)}$  accounting for both logarithmic and non-logarithmic contributions in the first loop. Repeating the same procedure for the gluon coefficient function, we obtain its first-loop value  $C_g^{(1)}$ . Apart from the trivial replacement  $S_1, S_2$  by  $\psi(\omega)$  according to Eq. (22),  $C_q^{(1)}$  and  $C_g^{(1)}$  differ from the NLO DGLAP coefficient functions  $C_{q,g}^{NLO \text{ DGLAP}}(\omega)$  by the treatment of  $\alpha_s$ :

$$C_q^{(1)}(\omega) = C_q^{NLO \text{ DGLAP}}(\omega)|_{\alpha_s \rightarrow A}, \quad C_g^{(1)}(\omega) = C_g^{NLO \text{ DGLAP}}(\omega)|_{\alpha_s \rightarrow A}, \quad (23)$$

with  $A$  being defined in Eq. (17). The two-loop expressions  $H_{ik}^{(2)}$  for the anomalous dimensions can be found quite similarly. They also can be obtained from the NLO DGLAP anomalous dimensions  $\gamma_{ik}^{NLO \text{ DGLAP}}(\omega)$  with expressing  $S_1, S_2$  through  $\psi(\omega)$  and replacing  $\alpha_s(Q^2)$  by  $A(\omega)$ :

$$H_{ik}^{(2)}(\omega) = \gamma_{ik}^{NLO \text{ DGLAP}}(\omega)|_{\alpha_s \rightarrow A}. \quad (24)$$

Explicit expressions for the NLO DGLAP coefficient functions and anomalous dimensions can be found e.g. in Ref. [11]. Obviously, the replacement  $\alpha_s(Q^2)$  by  $A(\omega)$  in Eqs. (23,24) makes possible to use  $C_q^{(1)}, C_g^{(1)}$  and  $H_{ik}^{(2)}$  at arbitrary  $Q^2$  in contrast to the DGLAP expressions for the coefficient functions and anomalous dimensions. Combining  $C_q^{(1)}, C_g^{(1)}$  and  $H_{ik}^{(2)}$  with Eqs. (9,11), we obtain the interpolation formulae equally valid for small and large  $\bar{x}$ . Indeed, the replacements  $H_{ik}$  by  $\tilde{H}_{ik}$  and  $C_{q,g}^{(\pm)}$  by  $\tilde{C}_{q,g}^{(\pm)}$  in Eq. (6) allow to extend the small- $\bar{x}$  formula Eq. (6) to arbitrary  $\bar{x}$ . The new coefficient functions  $\tilde{C}_{q,g}^{(\pm)}$  are defined as follows (the superscripts  $\pm$  are dropped here):

$$\tilde{C}_q = C_q + C_q^{(1)} - \Delta C_q, \quad \tilde{C}_g = C_g + C_g^{(1)} - \Delta C_g \quad (25)$$

where  $C_{q,g}$  are defined in Eq. (9),  $\Delta C_{q,g}$  are their perturbative first-loop expansions and  $C_q^{(1)}, C_g^{(1)}$  are given by Eq. (23). The definitions for new anomalous dimensions  $\tilde{H}_{ik}$  look quite similar:

$$\tilde{H}_{ik} = H_{ik} + H_{ik}^{(2)} - \Delta H_{ik} \quad (26)$$

where  $H_{ik}$  are introduced in Eq. (11),  $\Delta H_{ik}$  include the first and second terms of their perturbative expansions whereas  $H_{ik}^{(2)}$  are given by Eq. (24). Now, introducing  $\tilde{\Omega}_{(\pm)}$  according to Eq. (7), with  $\tilde{H}_{ik}$  in place of  $H_{ik}$ , we arrive at the expression describing  $g_1$  at arbitrary  $\bar{x}$  and  $Q^2$ :

$$\begin{aligned} g_1(\bar{x}, \bar{Q}^2) = & \frac{\langle e_q^2 \rangle}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^\omega \times \\ & \times \left[ \left( \tilde{C}_q^{(+)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\tilde{\Omega}_{(+)}} + \tilde{C}_q^{(-)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\tilde{\Omega}_{(-)}} \right) \delta q(\omega) + \right. \\ & \left. + \left( \tilde{C}_g^{(+)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\tilde{\Omega}_{(+)}} + \tilde{C}_g^{(-)}(\omega) \left( \frac{Q^2 + \mu^2}{\mu^2} \right)^{\tilde{\Omega}_{(-)}} \right) \delta g(\omega) \right]. \end{aligned} \quad (27)$$

When  $Q^2 \ll \mu^2$ , Eq. (27) can be expanded in the series in  $Q^2/\mu^2$ :

$$g_1(\bar{x}, \bar{Q}^2) \approx g_1(z) + (Q^2/\mu^2) \frac{\partial g_1(\bar{x}, \bar{Q}^2)}{\partial Q^2/\mu^2} + O((Q^2/\mu^2)^2) \quad (28)$$

where

$$g_1(z) = \frac{\langle e_q^2 \rangle}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{z}\right)^\omega \left[ \tilde{C}_q(\omega) \delta q + \tilde{C}_g(\omega) \delta g \right]. \quad (29)$$

We have denoted here

$$\tilde{C}_q = \tilde{C}_q^{(+)} + \tilde{C}_q^{(-)} = C_q + C_q^{(1)} - \Delta C_q, \quad \tilde{C}_g = \tilde{C}_g^{(+)} + \tilde{C}_g^{(-)} = C_g + C_g^{(1)} - \Delta C_g \quad (30)$$

and

$$\begin{aligned} C_q &= C_q^{(+)} + C_q^{(-)} = \frac{\omega(\omega - H_{gg})}{\omega^2 - \omega(H_{gg} + H_{qq}) + H_{qq}H_{gg} - H_{qg}H_{gq}}, \quad \Delta C_q = 1 + \frac{a_{qq}}{\omega^2}, \\ C_g &= C_g^{(+)} + C_g^{(-)} = \frac{\omega H_{gq}}{\omega^2 - \omega(H_{gg} + H_{qq}) + H_{qq}H_{gg} - H_{qg}H_{gq}}, \quad \Delta C_g = \frac{a_{gq}}{\omega^2}. \end{aligned} \quad (31)$$

#### IV. IMPLICATIONS FOR THE RECENT COMPASS DATA.

Eq. (28) shows explicitly that  $g_1$  practically does not depend on  $x$  in the region of Eq. (2). It perfectly agrees with the flat  $x$ -dependence of  $g_1$  observed experimentally in Ref. [1]. Such a dependence means that  $g_1$  in the COMPASS kinematic region (2) does not depend on the conventional variables  $x$  and  $Q^2$ . On the contrary, Eq. (28) predicts that the  $z$ -dependence of  $g_1$  is pretty far from being trivial. Let us notice here that  $z$  is inversely proportional to the standard variable  $\nu = w/(2M)$  measured in GeV, with  $M = 1$  GeV:

$$z = \left(\frac{\mu^2}{2M}\right) \frac{1}{\nu} \approx \frac{15}{\nu}, \quad (32)$$

so the region (5) covered in the COMPASS experiment corresponds to the  $\nu$ -region (in GeV)

$$15 \lesssim \nu \lesssim 150. \quad (33)$$

Obviously, a straightforward and unambiguous application of our description of  $g_1$  to the COMPASS experiment could be obtained just by fitting the COMPASS data on  $g_1(z)$ . Unfortunately, this is impossible because the COMPASS collaboration has not studied the  $z$ -dependence of  $g_1$ . Nevertheless, it is clear that Eq. (1) could be satisfied at any  $z$  in the region (5) only if there exists a strong correlation between  $\delta q$  and  $\delta g$  to compensate the difference between  $C_q$  and  $C_g$  explicitly given in Eq. (31). We think that the chance for such a correlation is very tiny, though strictly speaking this situation cannot be excluded. An alternative interpretation of the COMPASS result is to consider Eq. (1) as

$$\langle g_1(z) \rangle = 0. \quad (34)$$

where  $\langle g_1(z) \rangle$  is the average value of  $g_1$  observed by COMPASS. Obviously, in order to match Eq. (34),  $g_1(z)$  should acquire both positive and negative values in the region (5). For further investigations with Eq. (34) one should choose appropriate fits for the initial parton densities  $\delta q(z)$  and  $\delta g(z)$ . Such fits are practically absent in the literature. In Ref. [6] we suggested to approximate  $\delta q(z)$  and  $\delta g(z)$  at small  $z$  by constants to get a rough estimate. However,  $z$  in the COMPASS region (5) is not small, so we prefer to use a DGLAP-like set of fits:

$$\delta q(z) = N_q z(1-z)^3(1+3z), \quad \delta g(z) = N_g(1-z)^4(1+3z). \quad (35)$$

This set corresponds to the DGLAP-fits suggested in Ref. [3] but does not coincide with them. The difference is in the power factors  $z^a$  while the terms in the brackets in Eq. (35) and in Ref. [3] coincide ( $x$  in Ref. [3] is replaced by  $z$  in Eq. (35)). Indeed the fit for  $\delta q$  in Ref. [3] contains the singular power factor  $x^{-0.5}$  whereas the power factor for  $\delta g$  is  $x^{0.5}$ . In Ref. [12] we have proved that the role played by the singular terms  $x^{-a}$  in the DGLAP fits is to mimic the total resummation of  $\ln^k(1/x)$ . When the resummation is taken into account, such factors (namely the factor  $z^{-0.5}$  in  $\delta g$ ) does not make sense any longer and should be dropped. The same is obviously true when  $x$  is changed by  $z$ . So, extracting the singular factor  $x^{-a}$  from the fit in Ref. [3], we arrive at Eq. (35). Now it is easy to check that the fits (35) do not lead to a flat  $z$ -dependence for  $g_1$  and cannot keep  $g_1(z) = 0$  in the whole COMPASS region (2).

In more detail by substitution of Eq. (35) into Eq. (29) and performing the integration over  $\omega$  numerically, with fixed and positive  $N_q$  and varying the values of  $N_g$ , we plot our results in Fig. 1. By a close inspection of the various configurations shown, we can easily conclude that these fits could be compatible with Eq. (34) only if  $N_g > 0$  and  $N_g > N_q$ .

As the way of averaging  $g_1$  over  $z$  in the COMPASS data is unknown, we can try another possibility, approximating

$$\langle g_1(z) \rangle \approx g_1(\langle z \rangle) = 0, \quad (36)$$

where  $\langle z \rangle = 0.25$  (i.e.  $\langle \nu \rangle \approx 60$  GeV) is the mean value of  $z$  from the region (5). Then using Eqs. (29,35), keeping positive  $N_q$  and varying  $N_g$ . Figs. 1 suggest again that  $N_g$  are positive and  $N_g > N_q$ .

## V. CONCLUSION

In the present paper we have considered in detail the recent COMPASS data on  $g_1$ . These data first confirm our prediction in Ref.[7] that  $g_1$  at small  $Q^2$  does not depend on  $Q^2$  and  $x$ . Instead, we predict that  $g_1$  depends on the invariant energy  $w = 2pq$  and the experimental investigation of this dependence would allow to estimate the initial parton densities. Unfortunately, this information is absent in the present COMPASS data, so a reliable study of the initial parton densities cannot be done. However, we have suggested two possible interpretations, Eqs. (34) and (36), of the COMPASS result Eq. (1). Combining the LLA resummation with the explicit first-loop values of the coefficient functions and using the DGLAP-like parametrization (35) of the initial parton densities, we conclude that the data suggest rather positive than negative values of the initial gluon density. We remind that our analysis is tentative. More quantitative conclusions can be drawn only after an accurate experimental study of the  $z$ -dependence of  $g_1$  has been performed.

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- [1] COMPASS Collaboration (E.S. Ageev et al), *Spin asymmetry  $A_1(d)$  and the spin-dependent structure function  $g_1(d)$  of the deuteron at low values of  $x$  and  $Q^{*2}$* , *Phys. Lett. B* **647** (2007) 330.
  - [2] G. Altarelli and G. Parisi, *Asymptotic Freedom in Partonic Language*, *Nucl. Phys. B* **126** (1977) 298;  
V.N. Gribov and L.N. Lipatov, *Deep inelastic  $e p$  scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438;  
L.N.Lipatov, *The parton model and perturbation theory*, *Sov. J. Nucl. Phys.* **20** (1974) 94;  
Yu.L. Dokshitzer, *Calculation of the Structure Functions for Deep Inelastic Scattering and  $e^+ e^-$  Annihilation by Perturbation Theory in Quantum Chromodynamics (In Russian)*, *Sov. Phys. JETP* **46** (1977) 641.
  - [3] G. Altarelli, R.D. Ball, S. Forte and G. Ridolfi, *Determination of the Bjorken sum and strong coupling from polarized structure functions*, *Nucl. Phys. B* **496** (1997) 337;  
*Theoretical analysis of polarized structure functions*, *Acta Phys. Polon. B29* (1998) 1145.
  - [4] E. Leader, A.V. Sidorov and D.B. Stamenov, *Longitudinal polarized parton densities updated*, *Phys. Rev. D* **73** (2006) 034023.
  - [5] J. Blumlein and H. Botcher, *QCD analysis of polarized deep inelastic data and parton distributions*, *Nucl. Phys. B* **636** (2002) 225;  
M. Hirai et al., *Determination of polarized parton distribution functions and their uncertainties*, *Phys. Rev. D* **69** (2004) 054021.
  - [6] B.I. Ermolaev, M. Greco and S.I. Troyan, *Singlet structure function  $g(1)$  at small  $x$  and small  $Q^{*2}$* , *Eur. Phys. J. C* **50** (2007) 823.
  - [7] B.I. Ermolaev, M. Greco and S.I. Troyan, *Perturbative power  $Q^{*2}$ -corrections to the structure function  $g(1)$* , *Eur. Phys. J. C* **51** (2007) 859.
  - [8] B.I. Ermolaev, M. Greco and S.I. Troyan, *Intercepts of the nonsinglet structure functions*, *Nucl. Phys. B* **594** (2001) 71;  
*QCD running coupling effects for the nonsinglet structure functions at small  $x$* , *ibid.* **571** (2000) 137.
  - [9] B.I. Ermolaev, M. Greco and S.I. Troyan, *Running coupling effects for the singlet structure function  $g(1)$  at small  $x$* , *Phys. Lett. B* **579** (2004) 321.
  - [10] B. Badelek and J. Kwiecinski, *Analysis Of The Electroduction Structure Functions In The Low  $Q^{*2}$  Region Combining The Vector Meson Dominance And The Parton Model With Possible Scaling Violation*, *Z. Physik C* **43** (1989) 251;

*Low  $Q^{*2}$ , low  $x$  region in electroproduction: An Overview*, Rev. Mod. Phys. 681996445;  
*Unified description of the nonsinglet spin dependent structure function  $g_1$  incorporating Altarelli-Parisi evolution and the double logarithmic  $\ln^{*2}(1/x)$  effects at low  $x$* ,  
*Phys. Lett. B* **418** (1998) 229.

- [11] M. Gluck, E. Reya, M. Stratmann and W. Vogelsang, *Next-to-Leading Order Radiative Parton Model Analysis of Polarized Deep Inelastic Lepton Nucleon Scattering*, *Phys. Rev. D* **63** (1996) 4775.
- [12] B.I. Ermolaev, M. Greco and S.I. Troyan, *Non-singlet structure functions: Combining the leading logarithms resummation at small- $x$  with DGLAP*, *Phys. Lett. B* **622** (2005) 93.
- [13] B.I. Ermolaev, M. Greco and S.I. Troyan, *Treatment of the QCD coupling in high energy processes*, *Phys. Lett. B* **522** (2001) 57.

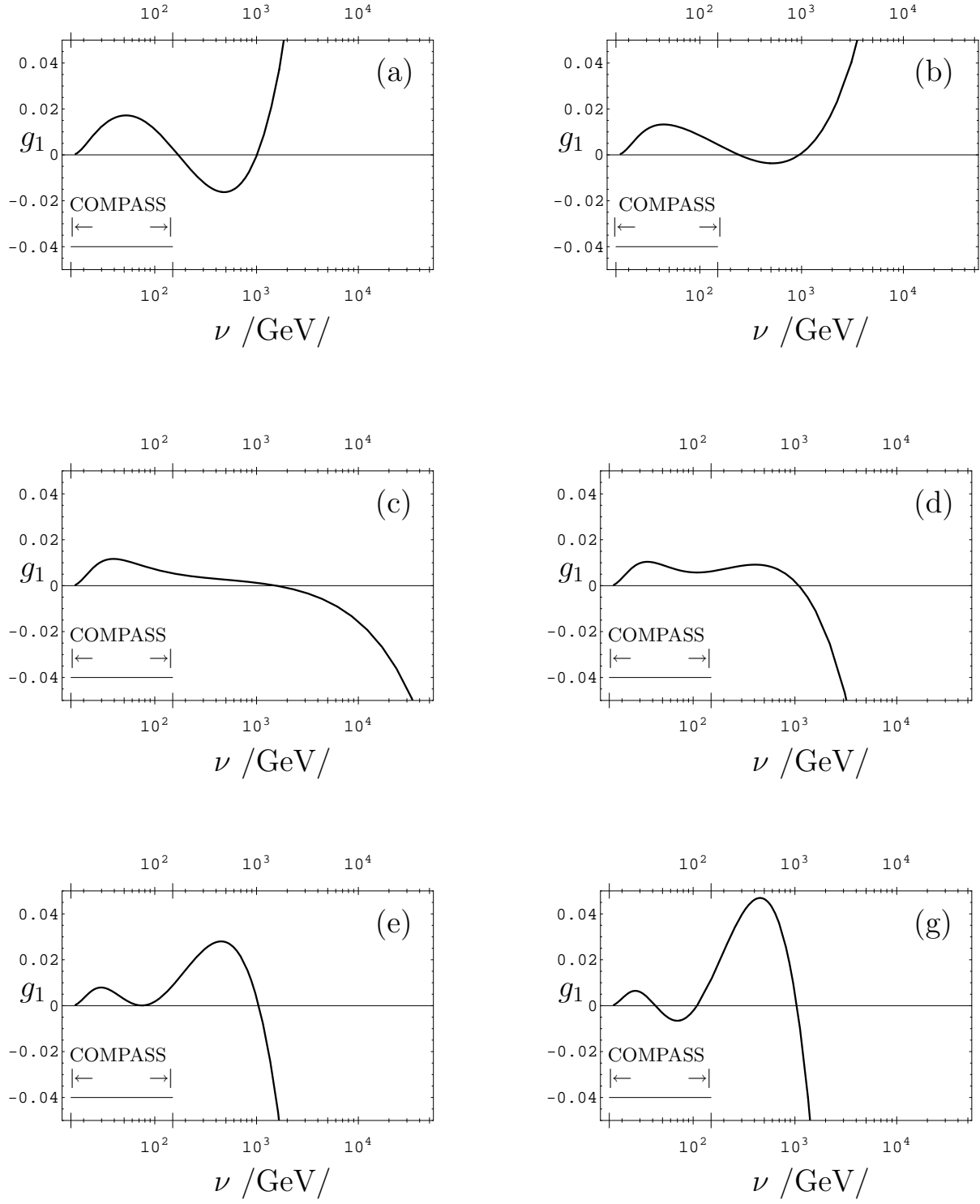


FIG. 1: The  $\nu$ -dependence of  $g_1(\nu)$ , with  $\delta q, \delta g$  defined in Eq. (35), for  $N_q = 0.5$  and different values of  $N_g$ : (a) -1.5, (b) -0.5, (c) 0, (d) 0.5, (e) 2, (g) 3.5; the COMPASS  $\nu$ -region corresponds to Eq. (33) .